APPENDIX D

Pedagogical Strategy

. . . as simple as possible, but not simpler.
attributed to A. Einstein

Physics first!
Anon.

The straightforward approach to teaching general relativity is to

1. develop the necessary mathematical concepts and tools,
2. motivate the Einstein equation and the requisite physical concepts,
3. solve the equation for the models of realistic physical situations, and
4. compare the predictions of the theory with experiment and observation.

The logic of this order is unassailable, and, by and large, it is the way the theory is presented in the classic expositions mentioned in the bibliography, as well as many excellent introductory texts. However, following this order in the limited time that is typically available and appropriate for a basic introductory course is difficult. There is a considerable body of beautiful, powerful, and straightforward mathematics that is necessary. But developing it takes time. Similarly, solving the nonlinear Einstein equation in any realistic situation can be a lengthy exercise. The length available for an introductory course is often not sufficient to present the subject in this logical way and also discuss its important applications. This book introduces general relativity in a different order. In this Appendix we present some pedagogical principles on which the present text is constructed.

D.1 Pedagogical Principles

Explore First, Derive Later

The simplest physically relevant solutions of the Einstein equation are presented *first*, without derivation, as spacetimes whose observational consequences are to be explored by the study of the motion of test particles and light rays in them. This brings the student to the physical phenomena as quickly as possible. It is the part of the subject most directly connected to classical mechanics, and requires the minimum of new mathematical ideas. Later the Einstein equation is introduced and solved to show where these geometries originate. Readers who have time to work through the entire text should understand both the important solutions and their origin. But those who stop earlier will at least have understood some of the basic phenomena for which curved spacetime is important.

Only the Simplest Examples

The simplest solutions of the Einstein equation are the most physically relevant. The Sun is approximately spherical, the universe is approximately homogeneous and isotropic, and detectable gravitational waves are weak and approximately planar. Only the simplest physically relevant spacetimes of general relativity are presented. Thus we discuss black holes with mass and angular momentum but not charge, spherical gravitational collapse quantitatively but nonspherical collapse only qualitatively, homogeneous, isotropic cosmologies but not anisotropic ones, weak gravitational waves in flat spacetime but not nonlinear waves or waves in curved spacetime, and spherical stars but not rotating ones.

Introduce New Math Only As Necessary

Mathematical ideas beyond those in the usual advanced calculus toolkit are introduced only as needed. Only a few additional tools are needed to understand a spacetime geometry and explore it through the motion of test particles and light rays. The basic concepts of metric, four-vector, and geodesics largely suffice. These are introduced in various chapters at the start of Parts 1 and 2 and are sufficient for all the development there. It is not necessary, for example, to develop a general theory of tensors in Parts 1 and 2, because only one tensor—the metric—is used. Tensors and the covariant derivative are introduced in Chapter 20. Quantitative measures of curvature are introduced in Chapter 21 as a prerequisite to understanding the Einstein equation.

Stress Physical Phenomena and Their Connection to Experiment and Observation

The Global Positioning System, the orbits of planets and light in the solar system, X-ray binaries, active galactic nuclei, neutron stars, gravitational lensing, gravitational waves, the large-scale structure of the universe, and the big bang are just some of the phenomena in the universe for which relativistic gravity is important. This book stresses the growing connection between general relativity and experiment and observation. Astrophysics and cosmology are home to many of these applications. However, *this is not a text on astrophysics or cosmology.* The connection between theory and observation is typically made by way of only the simplest type of model, and then often only in a qualitative way.

Classic Experiments but Not an Overview of Experiment

No contemporary exposition of general relativity would be complete without describing its experimental confirmation and application to astrophysics. But the inevitable downside to any discussion of experiment and observation is that it will become quickly dated. That is especially the case in gravitational physics, where the domain of application is growing rapidly at the time of writing and will grow even faster when the gravitational wave detectors now under construction come on line. For this reason the author has not tried to write an overview of the experimental situation, nor necessarily included the latest data, but rather has used classic examples that illustrate the basic methods.

D.2 Organization

Prerequisites

The main prerequisite is the introductory mechanics course that is typically a standard part of any undergraduate major in physics. Especially important are a grounding in the general principles of mechanics, conservation laws, orbits in the central force problem, and Lagrangian mechanics. An introduction to the variational principle for mechanics will be helpful although an abbreviated discussion is given in Chapter 3. Similarly an introduction to special relativity would be helpful but the discussion in Chapters 4–5 is self-contained. There are passing references to Maxwell's equations, and there are elementary applications of electromagnetism in the boxes, but a detailed course in the subject is not a prerequisite to tackling the main text.

The Three Parts

The book is divided into three parts. Part 1 introduces the idea that gravity is geometry and reviews the basic parts of Newtonian and special relativistic mechanics that are relevant for general relativity. Part 2 introduces the basic ideas of general relativity and then focuses on understanding the simplest black hole, cosmological, and gravitational wave spacetimes through a study of the motion of test particles and light rays in them. These geometries are presented and analyzed, not derived. They are derived in Part 3 after the mathematics of curvature and the Einstein equation are introduced. Part 3 goes on to give an elementary discussion of the production of gravitational waves and relativistic stars for which the Einstein equation is essential. Within each part the order of the topics is roughly by increasing sophistication—either of mathematical detail or physical concept or both.

Boxes

The discussion in the boxes is intended to extend and illustrate the basic ideas in the main text. Sometimes a box concerns a related idea (such as Penrose diagrams), sometimes a relevant experiment (such as a modern Michelson–Morley experiment), and sometimes an introduction to a complex phenomenon in which general relativity plays an important role (such as the electromagnetic extraction of energy from rotating black holes). Some of these extensions require modest parts of physics beyond the basic mechanics assumed for most of the main text. The discussion in such cases is typically more qualitative and abbreviated than the standard typical of the main text. The aim of the boxes is not to achieve an in-depth understanding of the subjects treated, but to illustrate some of the ramifications of the main development briefly and qualitatively. Depending on their preparation, students will find some boxes more difficult to understand than others, but it is not necessary to understand any box to understand the main text.

Mathematica Notebooks

Analyzing even the simplest of physical situations in general relativity can sometimes require messy algebra, or lead to differential equations lacking elementary closed-form solutions. To help with this, the following *Mathematica* notebooks are provided on the book website which do some standard algebra and solve some of the most important differential equations.

- Christoffel Symbols and Geodesic Equation
- Shape of Orbits in the Schwarzschild Geometry
- Friedman-Robertson-Walker Cosmological Models
- Curvature and the Einstein Equation

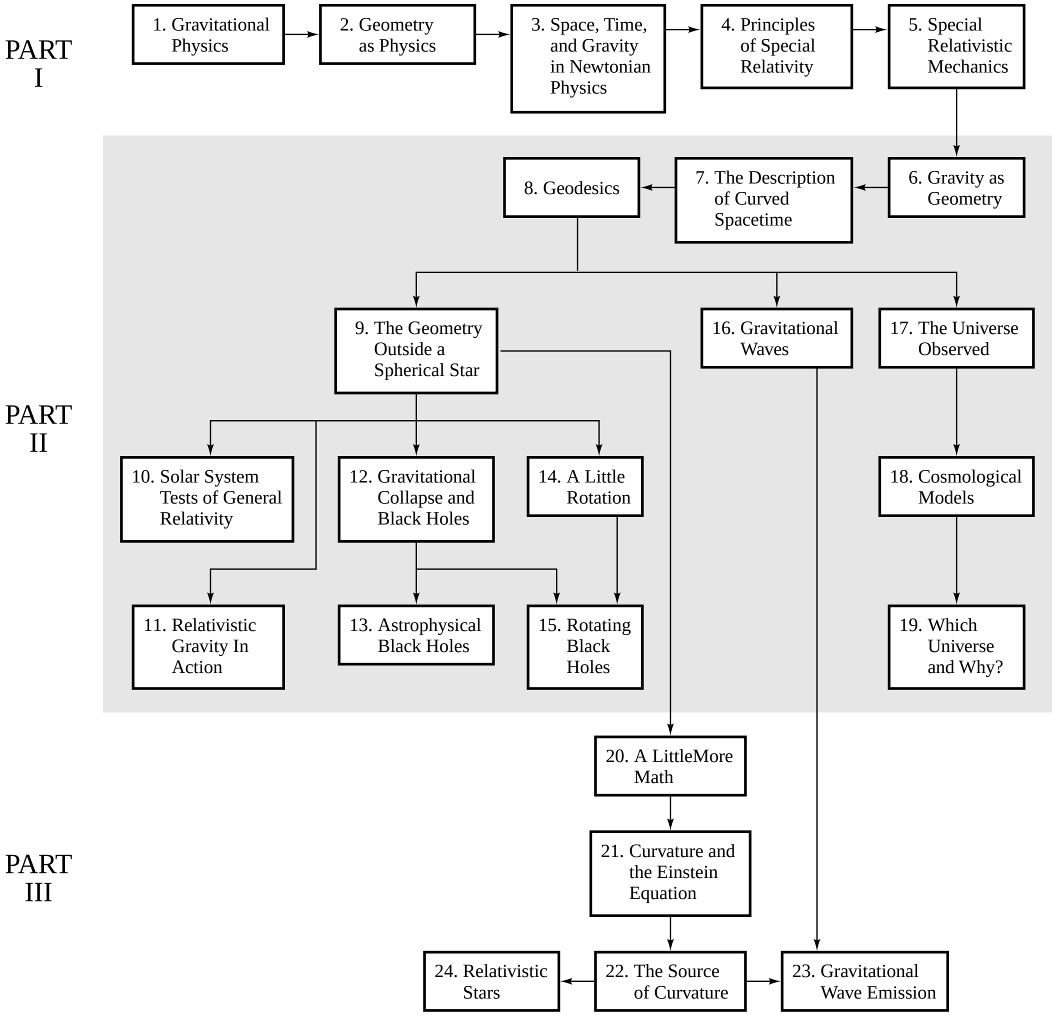
Web Supplements

Some conceptually simple results require lengthy derivations that tend to interrupt the main development. Conventionally these would be relegated to appendices. But to keep the book to a managable length these are housed in the book website.

D.3 Constructing Courses

The text contains more material than can be reasonably covered in a one-quarter (∼ 30 hour lectures) or a one-semester (∼ 45 hour lectures) course. A variety of course plans can therefore be constructed by selecting chapters, or parts of them, in various ways. The following chart shows how the various chapters depend on each other. Student preparation will determine where to start in the first part (Chapters 1 through 5) or how quickly to cover them. Chapters 6–9 introduce some basic ideas and techniques of general relativity. Any selection of later chapters that includes the earlier ones on which they depend could in principle form the basis of a course. For example, a focus on black holes would include Chapters 12–15. A focus on gravitational waves might include Chapters 16, 20–23. The author's own quarter course typically covers Chapters 1–10, 12–13, and then Chapters 17–19 on cosmology or Chapters 20–21 introducing the Einstein equation, depending on class interest.

The author has several times employed the text as the basis for an introductory graduate course. It works well for students who are seeing the general relativity for the first time, are more interested in applications than the general framework, or if there is limited time.


PART I
1. Gravitational Physics
2. Geometry as Physics
3. Space, Time, and Gravity in Newtonian Physics
4. Principles of Special Relativity
5. Special Relativistic Mechanics
PART II
8. Geodesics
7. The Description of Curved Spacetime
6. Gravity as Geometry
9. The Geometry Outside a Spherical Star
16. Gravitational Waves
17. The Universe Observed
10. Solar System Tests of General Relativity
12. Gravitational Collapse and Black Holes
14. A Little Rotation
18. Cosmological Models
11. Relativistic Gravity In Action
13. Astrophysical Black Holes
15. Rotating Black Holes
19. Which Universe and Why?
PART III
20. A LittleMore Math
21. Curvature and the Einstein Equation
24. Relativistic Stars
22. The Source of Curvature
23. Gravitational Wave Emission