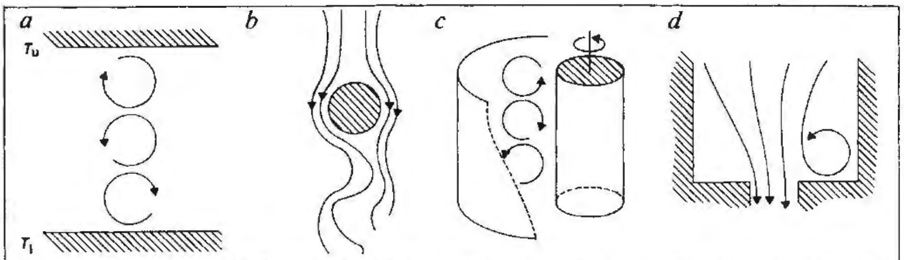

Symmetry-breaking fluid flows. *a*, Thermal convection currents between two plates at different temperature ($T_u > T_l$). *b*, Eddy pattern around a cylindrical obstruction. *c*, Taylor vortices in a plane perpendicular to that of rotating central cylinder. *d*, Asymmetrical flow pattern of a non-newtonian fluid undergoing constricted flow.

pattern must be smaller than the smallest dimension of the model cell to avoid distortions caused by the periodic boundary conditions. Therefore, the size of the cell and the number of particles must increase with the Reynolds and Rayleigh numbers. The computer time increases at least in proportion to the number of particles considered, and the real time during which the particles are followed is typically less than 1 nanosecond.

Despite these temporary limitations, molecular dynamics offers the ability, on one level, to generate molecularly based constitutive equations as a basis for macroscopic-flow calculations using finite-difference or cellular-automata algorithms (see Wolfram, S. *Nature* **311**, 419; 1984). In the long term, it could be used to investigate these phenomena directly. □

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Nonlinear dynamics

Chaos is good news for physics

Tomas Bohr and Predrag Cvitanović

In a recent paper, E.G. Gwinn and R.M. Westervelt (*Phys. Rev. Lett.* **59**, 157–160; 1987) explore a route from regular to chaotic behaviour in electronic transport in cooled p-type germanium. The same route to chaos has also been explored by A. Libchaber and co-workers in a very different physical system: the convective flow of mercury (Jensen, M.H. *et al. Phys. Rev. Lett.* **55**, 2798–2801; 1985). The new experiment is interesting as a study of semiconductor physics and for its many technological ramifications; but both experiments illustrate the high precision with which experimentalists can now test the theory of transitions to chaos.

Chaos has become a cover name for an active branch of physics, describing the highly irregular, unpredictable behaviour that occurs in most deterministic, nonlinear dynamical systems. Many mathematical concepts dating back to the last century — and, until recently, regarded by physicists as irrelevant to the description of natural phenomena — have suddenly become important. Notions such as the 'Hausdorff dimension' and 'fractals' have replaced Euclid's dimensions and straight lines, because schoolbook geometry is of little use when confronted with the bewildering complexity generated by nonlinear dynamical systems. The work of B.B. Mandelbrot and especially his book *The Fractal Geometry of Nature* (Freeman, New York, 1982) has been instrumental in turning physicists' attention to such fractal structures and introducing the relevant mathematical concepts.

A decade ago, M.J. Feigenbaum (*J. stat. Phys.* **21**, 669–706; 1979) conjectured that many physical systems should make the transition from regular (periodic) to chaotic motion in a qualitatively and quantitatively universal fashion, through sequences of period doublings. His approach was very unconventional: instead of modelling a realistic dynamical system he used a computer to iterate a simple map. The regime where chaos occurs is quite inaccessible by standard approximation techniques; but the map reduces the problem to its essence, making it possible to follow the strongly nonlinear behaviour with a minimum of effort and still extract universal quantities. The theory has been confirmed experimentally in many physical systems, ranging from convective flows in liquids to cardiac arrhythmias in chicken hearts. Original articles are collected in two reprint collections: Hao Bai-Lin (ed.), *Chaos* (World Scientific, Singapore, 1984); and *Universality in Chaos* (Hilger, Bristol, 1984; edited by P.C.).

In their experiment, Gwinn and Westervelt apply a time-varying voltage to a crystal of p-type germanium and record the current passing through it. The system has an oscillatory instability so that even a static (d.c.) voltage larger than some threshold value induces a periodic oscillation of the current, with an extremely stable frequency. When the additional time-varying component is added to the voltage, the two frequencies compete, and if the amplitude of the driving frequency is large enough, chaotic behaviour develops. The interaction of pairs of frequencies is of considerable theoretical interest because of the generality of the phenomenon. As the energy input into a dissipative dynamical system is increased, typically first one and then another intrinsic mode of the system is excited, and their interaction can give rise to chaos. Competing modes usually give rise to mode-lockings: the frequencies adjnst slightly to fall into step, making their ratio a rational number. In that case the response is periodic; an irrational ratio corresponds to quasiperiodic behaviour — the motion never quite repeats itself.

The aim of the experiment was to test the theory of the transition from quasiperiodicity to chaos. Mode-lockings (which have interesting properties of their own) are avoided by tuning the external frequency so that its ratio to the internal frequency equalled the golden mean, $(\sqrt{5}-1)/2$. The choice of this ratio does not represent a return to mediaeval alchemy, but is dictated by number theory: the golden mean belongs to a family of irrational numbers for which it is hardest to give good rational approximants. As experimental measurements have limited accuracy, physicists usually do not expect number-theoretic subtleties, such as how irrational a number is, to be of any physical interest. In the theory of transitions to chaos, however, the starting point is the enumeration of asymptotic motions of a dynamical system, and it is through this enumeration that number theory enters and comes to have a central role.

The output of the experiment is the time variation of the current, the full record of which contains much superfluous

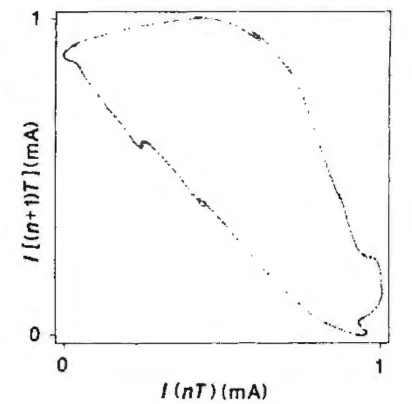

Fig. 1 The strange attractor at the onset of chaos. The current (a constant current of 5 mA has been deducted) is plotted for each cycle of the external drive. T and n, respectively, are the period and number of drive cycles and the current at cycle $n+1$ is plotted against the value at cycle n. (From Gwinn, E.G. & Westervelt, R.M. *Phys. Rev. Lett.* **59**, 157–160; 1987.)

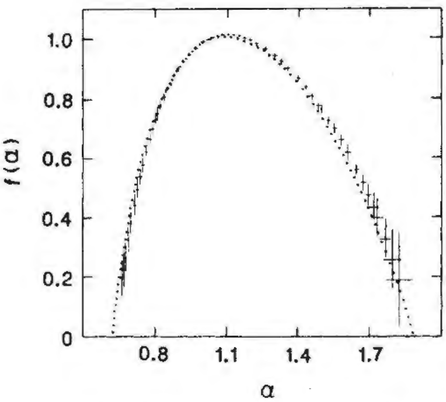

Fig. 2 The function of $f(\alpha)$ quantifying how often a given scaling index or pointwise dimension occurs on the attractor. The error bars indicate the standard deviation of the mean of three data sets. The universal circle-map prediction is shown as a dotted curve. (From Gwinn, E.G. & Westervelt, R.M. *Phys. Rev. Lett.* **59**, 157–160; 1987.)

information. Previously, the results would have been presented as a frequency power spectrum. Today the data (50,000 data points would be typical) are manipulated in the way that a theorist analyses a numerical simulation. First, the system is reduced from a continuous time recording to a discrete series of stroboscopic flashes. This method, devised a century ago by H. Poincaré, makes it possible to survey the dynamics visually. The experimental Poincaré map is shown in Fig. 1. The value of the current at each period of the external drive is plotted against its value at the next period. (The first hundred or so cycles, the transients, are not plotted.) The fact that the response is quasiperiodic implies that the dots on the figure would eventually fill up some closed curve. If the transients had been shown on Fig. 1, we would see points initially far away rapidly approaching that curve, which is therefore an 'attractor'. Precisely at the onset of chaos — which was reached in the experiment by varying the amplitude of the drive — the kinky structure visible on the figure emerges. It is called a 'strange attractor' because of the unusual way points are distributed on it and this distribution contains information about universal features of the transition to chaos.

The strangeness of the attractor can be probed by looking at the distribution of points around some reference point. For a reference point, P, on the attractor we define $N_P(r)$ as the number of neighbours within distance r along the attractor. On a usual (non-strange) quasiperiodic attractor, the points are distributed smoothly; a segment of the attractor looks like a line. Thus, $N_P(r)$ scales with r as $N_P(r) \propto r$ for any point P. For the strange attractor in the figure this is not necessarily true. If we pick a point at random, the theory predicts $N_P(r) \propto r^{\alpha}$ with α varying between 0.6326 . . . and 1.8980 . . ., and the point P is said to have the pointwise dimension α.

A powerful formalism for confronting the experiment with the predictions of the theory was developed recently by T.C. Halsey *et al.* (*Phys. Rev.* **A33**, 1141–1151; 1986). They look at the attractor as a superposition of 'subfractals'; namely, for each α, the set of points with pointwise dimension α. A function $f(\alpha)$ is introduced to quantify how often a given value α appears on the attractor — more precisely, $f(\alpha)$ is the Hausdorff dimension of the set of points with pointwise dimension α. Figure 2 shows this curve for the theory and the experiment. The dots are the theoretical prediction computed by iterating a so-called circle-map. The crosses represent $f(\alpha)$ calculated from the experimental attractor shown in Fig. 1. Given that there are no adjustable parameters, the agreement between the theory and the experiment is remarkable. (The error bars at the edges of the α-interval come from the sparseness of points contributing in those limits; here the finiteness of the data sets and noise play a large part.)

Theory and experiment thus interact in a fruitful way, and it is fascinating that the fractal properties of strange attractors can be measured with such precision in physical systems. Theoretically the most striking fact, not anticipated until a decade ago, is that the dynamical systems approaching chaos do so in a universal fashion. Other fields of physics, notably those concerned with growth or aggregation, reveal all kinds of fractal behaviour, but it is not yet clear what kind of universality (if any) to expect there. □

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Intermediate filaments

Looking for a function

Benjamin Geiger

STUDIES on the molecular properties of the cytoskeleton are largely motivated by the desire to understand the functions of these cytoplasmic filaments in cells and tissues. Physiological studies, morphological observations and biochemical characterization of proteins in the cytoskeleton have allowed molecular models, albeit tentative, of the possible cellular activities in which the various classes of filaments take part. These models, despite their tendency to be oversimplified, have contributed a great deal to current concepts of mechanisms of cell motility, mitosis, transcellular transport, adhesion, modulation of membrane activity and cellular morphogenisis. Now, however, experimental work[1-4] is providing a test for many of these ideas about the physiological functions of intermediate filaments, one class of cytoskeletal filaments.

Until recently, most information has come from the other two classes of cytoskeletal filaments, microfilaments and microtubules, which, together with batteries of associated proteins, have been extensively characterized. Moreover, the availability of excellent and well-studied model systems such as the contractile unit of skeletal muscle or the interaction of dynein with microtubules in cilia and flagellae have provided clues about the cytoplasmic activities in which actin and tubulin are involved.

The structure–function relationships of the third cytoskeletal network, the intermediate filaments, are less well characterized. Despite the fact that the primary structure of many intermediate-filament subunits is known and their cellular distribution extensively documented, only limited molecular information has so far been available on their behaviour *in vivo*. In the absence of specific data, it has been suggested that intermediate filaments are involved in mechanical integration of cytoplasmic space[5] or in a skeletal framework of the cytomatrix (see refs. 6,7 for reviews). Well-controlled experiments relating their molecular properties to specific cytoplasmic events are, however, still needed to understand their functions in more detail.

Inagaki *et al.*[1] recently investigated specific phosphorylation events in modulating the assembly of vimentin molecules into intermediate filaments. These authors show that vimentin is an excellent *in vitro* substrate for protein kinase C and cyclic AMP-dependent protein kinase, but not of several other kinases. Moreover, phosphorylation by the cyclic-AMP-dependent kinase induces the dramatic disassembly of vimentin filaments. Analysis of tryptic phosphopeptide maps indicates that the sites of phosphorylation with the two kinases are distinct, and the authors suggest that a single, site-specific phosphorylation of vimentin (as well as its

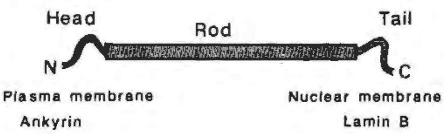

Fig. 1 Intermediate-filament subunit (vimentin or desmin) presenting binding sites for plasma-membrane-associated ankyrin at the amino-terminal head domain, and binding sites for lamin B at the carboxy-terminal tail region.