Exercises

three-disk@3-disk!simulator pinball!simulator three-disk@3-disk!simulator

maybe use different letters for rotates and continuous-time escape rates?

Exercise 6.1 **A pinball simulator.** *Implement the disk → disk maps to compute a trajectory of a pinball for a given starting point, and a given $R{:}a$ = (center-to-center distance):(disk radius) ratio for a 3-disk system. As this requires only computation of intersections of lines and circles together with specular reflections, implementation should be within reach of a high-school student. Please start working on this program now; it will be continually expanded in chapters to come, incorporating the Jacobian calculations, Newton root–finding, and so on.*

Fast code will use elementary geometry (only one $\sqrt{\cdots}$ per iteration, rest are multiplications) and eschew trigonometric functions. Provide a graphic display of the trajectories and of the Poincaré section iterates. To be able to compare with the numerical results of coming chapters, work with $R{:}a = 6$ and/or 2.5 values. Draw the correct versions of figure 1.8 or figure 11.4 for $R{:}a = 2.5$ and/or 6.

Exercise 6.2 **Trapped orbits.** *Shoot 100,000 trajectories from one of the disks, and trace out the strips of figure 1.8 for various $R{:}a$ by color coding the initial points in the Poincaré section by the number of bounces preceeding their escape. Try also $R{:}a = 6{:}1$, though that might be too thin and require some magnification. The initial conditions can be randomly chosen, but need not - actually a clearer picture is obtained by systematic scan through regions of interest.*

Include fig. 15 from LLNN.

Exercise 6.3 **Pinball stability.** *Add to your exercise 6.1 pinball simulator a routine that computes the the [2×2] Jacobian matrix. To be able to compare with the numerical results of coming chapters, work with $R{:}a = 6$ and/or 2.5 values.*

Exercise 6.4 **Stadium billiard.** *Consider the* Bunimovich stadium *[6.8, 37.65] defined in figure 6.1. The Jacobian matrix associated with the reflection is given by (6.10). Here we take $\rho_k = -1$ for the semicircle sections of the boundary, and $\cos\phi_k$ remains constant for all bounces in a rotation sequence. The time of flight between two semicircle bounces is $\tau_k = 2\cos\phi_k$. The Jacobian matrix of one semicircle reflection folowed by the flight to the next bounce is*

$$\mathbf{J} = (-1)\begin{pmatrix} 1 & 2\cos\phi_k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/\cos\phi_k & 1 \end{pmatrix} = (-1)\begin{pmatrix} -3 & 2\cos\phi_k \\ 2/\cos\phi_k & 1 \end{pmatrix}.$$

*A shift must always be followed by $k = 1, 2, 3, \cdots$ bounces along a semicircle, hence the natural symbolic dynamics for this problem is n-ary, with the corresponding Jacobian matrix given by shear (*ie. *the eigenvalues remain equal to 1 throughout the whole rotation), and k bounces inside a circle lead to*

$$\mathbf{J}^k = (-1)^k\begin{pmatrix} -2k-1 & 2k\cos\phi \\ 2k/\cos\phi & 2k-1 \end{pmatrix}. \tag{6.21}$$

compare to Jonas

three-disk@3-disk!simulator
three-disk@3-disk!phase space
repeller!3-disk
phase space!3-disk

The Jacobian matrix of a cycle p of length n_p is given by

$$\mathbf{J}_p = (-1)^{\sum n_k} \prod_{k=1}^{n_p} \begin{pmatrix} 1 & \tau_k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ n_k r_k & 1 \end{pmatrix} . \tag{6.22}$$

whole formula wrong

append Jonas re here

Find Woytkowski's purely geometric description

Adopt your pinball simulator to the Bunimovich stadium.

⇓PRELIMINARY

Exercise 6.5 Sinai-Bunimovich curvature. *Make an exercise relating κ and* **J**.

⇑PRELIMINARY

Exercise 6.6 A test of your pinball simulator. *Test your exercise 6.3 pinball simulator by computing numerically cycle stabilities by tracking distances to nearby orbits. Compare your result with the exact analytic formulas of exercise 8.1 and 8.2.*

⇓PRELIMINARY

Exercise 6.7 3-disk repeller fit. *Show by numerical comparison that the $R{:}a = 6{:}1$ 3-disk repeller has a rather thin repeller, and can be roughly fit with $a \approx 20$ in (??).*

⇑PRELIMINARY

Exercise 6.8 Birkhoff coordinates. *Prove that the Birkhoff coordinates are phase-space volume preserving. Hint: compute the determinant of (6.11).*

migrate the hint into solutions