three-disk@3-disk!cycle!analytically
stability!exact

Exercises

Exercise 8.1 **Fundamental domain fixed points.** *Use the formula (6.11) for billiard Jacobian matrix to compute the periods T_p and the expanding eigenvalues Λ_p of the fundamental domain $\overline{0}$ (the 2-cycle of the complete 3-disk space) and $\overline{1}$ (the 3-cycle of the complete 3-disk space) fixed points:*

$$\begin{array}{c|cc} & T_p & \Lambda_p \\ \hline \overline{0}: & R-2 & R-1+R\sqrt{1-2/R} \\ \overline{1}: & R-\sqrt{3} & -\frac{2R}{\sqrt{3}}+1-\frac{2R}{\sqrt{3}}\sqrt{1-\sqrt{3}/R} \end{array} \tag{8.21}$$

We have set the disk radius to $a = 1$.

Exercise 8.2 **Fundamental domain 2-cycle.** *Verify that for the $\overline{10}$-cycle the cycle length and the trace of the Jacobian matrix are given by*

$$\begin{aligned} L_{10} &= 2\sqrt{R^2-\sqrt{3}R+1}-2, \\ \mathrm{tr}\,\mathbf{J}_{10} &= 2L_{10}+2+\frac{1}{2}\frac{L_{10}(L_{10}+2)^2}{\sqrt{3}R/2-1}\,. \end{aligned} \tag{8.22}$$

recheck - fixed up by Per

The $\overline{10}$-cycle is drawn in figure 11.6. The unstable eigenvalue Λ_{10} follows from (4.18).

Exercise 8.3 **A limit cycle with analytic stability exponent** *There are only two examples of flows for which the stability eigenvalues can be evaluated analytically. One example is the 2-d flow*

$$\dot{q} = p + q(1-q^2-p^2)\,, \qquad \dot{p} = -q + p(1-q^2-p^2)\,. \tag{8.23}$$

Determine all periodic solutions of this flow, and determine analytically their stability exponents. Hint: go to polar coordinates $(q,p) = (r\cos\theta, r\sin\theta)$.

G. Bard Ermentrout