Rossler@Rössler system
map!contracting
map!dissipative
contracting!map
dissipative!map

Exercises

Exercise 4.1 Trace-log of a matrix. *Prove that*

$$\det M = e^{\mathrm{tr}\,\ln M} .$$

for an arbitrary finite dimensional matrix M.

Exercise 4.2 Stability, diagonal case. *Verify the relation (4.16)*

$$\mathbf{J}^t = e^{t\mathbf{A}} = \mathbf{U}^{-1} e^{t\mathbf{A}_D} \mathbf{U} , \quad \text{where} \quad \mathbf{A}_D = \mathbf{U}\mathbf{A}\mathbf{U}^{-1} .$$

Exercise 4.3 Topology of the Rössler flow. *(continuation of exercise 3.1)*

(a) *Show that equation $|\det(A - \lambda \mathbf{1})| = 0$ for Rössler system in the notation of exercise 2.18 can be written as*

$$\lambda^3 + \lambda^2 c\,(p^{\mp} - \epsilon) + \lambda(p^{\pm}/\epsilon + 1 - c^2 \epsilon p^{\mp}) \mp c\sqrt{D} = 0 \qquad (4.42)$$

(b) *Solve (4.42) for eigenvalues $\lambda^{\pm}$ for each equilibrium point as an expansion in powers of ϵ. Derive*

$$\begin{array}{ll} \lambda_1^- &= -c + \epsilon c/(c^2+1) + o(\epsilon) \\ \lambda_2^- &= \epsilon c^3/[2(c^2+1)] + o(\epsilon^2) \\ \theta_2^- &= 1 + \epsilon/[2(c^2+1)] + o(\epsilon) \\ \lambda_1^+ &= c\epsilon(1-\epsilon) + o(\epsilon^3) \\ \lambda_2^+ &= -\epsilon^5 c^2/2 + o(\epsilon^6) \\ \theta_2^+ &= \sqrt{1+1/\epsilon}\,(1+o(\epsilon)) \end{array} \qquad (4.43)$$

Compare with exact eigenvalues. What are dynamical implications of the extravagant value of λ_1^-?

(continued as exercise 4.3) *(Rytis Paškauskas)*

Exercise 4.4 A contracting baker's map. *Consider a contracting (or "dissipative") baker's map, acting on a unit square $[0,1]^2 = [0,1] \times [0,1]$, defined by*

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} x_n/3 \\ 2y_n \end{pmatrix} \quad y_n \le 1/2$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} x_n/3 + 1/2 \\ 2y_n - 1 \end{pmatrix} \quad y_n > 1/2$$

This map shrinks strips by a factor of 1/3 in the x-direction, and then stretches (and folds) them by a factor of 2 in the y-direction.

By how much does the phase space volume contract fo one iteration of the map? ⇓PRELIMINARY

(continued in exercise 11.4) ⇑PRELIMINARY