Solitons in nonlinear optics

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Abstract: A review on the application of soliton theory and soliton-type methods in nonlinear optics is given. We start with reviewing the most popular optical solitons: solitons in an optical wave guide and the 2π-pulses of self-induced transparency (SIT). Then we study the dynamics of stimulated Raman scattering (SRS) and two-photon propagation (TPP, i.e., the combination of two–photon absorption and emission), and we point out structural relations between SRS, TPP, degenerate TPP, SIT, three-wave interaction and second-harmonics generation.

Introduction

Solitons and, more generally, methods related to soliton theory have found a rich variety of applications in optics and, vice versa, a good deal of stimulation for soliton theory originates from nonlinear optics.

This printed contribution differs from the oral version. Instead of filling pages with formulae and derivations which can easily be found in the literature here I will give a short systematic overview together with a list of references. For nonresonant interaction processes only optical fields appear as dynamical variables while the structural properties of matter are hidden in nonlinear susceptibilities or coupling constants. In first- or higher-order resonant interaction processes, however, there is a dynamical interaction between matter and field. Thus nonlinear optical processes may be classified by simply counting the number of participating atomic levels as well as the number of optical waves. Here we consider the dynamics in one space dimension only together with time or, occasionally, stationary processes in two space dimensions.

1. One wave

The propagation of a quasi-monochromatic wave in a medium with Kerr nonlinearity is described by the nonlinear Schrödinger equation [1], and the same holds for a pulse propagating in an optical fiber [2]. Solitons in optical fibers were experimentally found at first by Mollenauer et al. [3]. In a planar waveguide there are *spatial solitons* in two space dimensions [5] which are described by the nonlinear Schrödinger equation as well. Solitons in fibers and in planar waveguides are very promising to become applied in optical communications [4].

2. Three waves

For the interaction of three fields with three different group velocities a Linear System with 3×3 matrices was found [6, 7]. The theory essentially simplifies when two velocities coincide. If two frequencies coincide we arrive at second harmonics generation (SHG). For SHG (with two different group velocities) a linear system exists [8], but there are no solitons. When restricted to real amplitudes SHG reduces to the Liouville equation [9] which is C-integrable, i.e., *integrable by change of variables.*

3. Two levels and one wave

One wave propagating in a resonant two-level system leads to *self-induced transparency* (SIT) [10, 11, 12]. In highest idealisation it is described by the old famous sine-Gordon equation. The integrability by inverse scattering or spectral transform method (for short: S-integrability) also holds for the more general Maxwell–Bloch equations including phase modulations and inhomogeneous broadening [13]. The same dynamical system when starting from fully excited atoms describes an amplifier or — when there is no input field — superfluorescence [14–16]. A review on SIT is given in [17].

4. Two levels and two waves

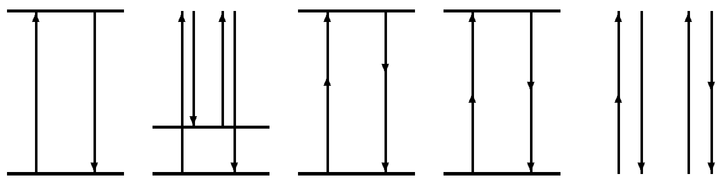
Now let two waves with carrier frequencies ω_1, ω_2 propagate through a medium consisting of two-level atoms with energy separation ΔE and assume that a second-order resonance $\omega_1 \pm \omega_2 = \Delta E/\hbar$ is fulfilled. For "−" this is *stimulated Raman scattering* (SRS), and for "+" it is *two-photon propagation* (TPP), i.e., two-photon absorption and emission [18–21]. The corresponding Linear System as the base for the inverse scattering method was found [22, 23] and N-soliton formulae were established [24]. For TPP with $\omega_1 = \omega_2$ we arrive at degenerate TPP or DTPP, sometimes called two-photon self-induced transparency [25, 26]. A *soliton* in SRS is a pulse at the pump frequency propagating within a long pulse at the Stokes frequency. For SRS restricted to low excitation of the Raman level soliton solutions were established by Chu and Scott [27] and experimentally found by Drühl et al. [28]. Such low excitation SRS is a three-wave interaction that is mathematically equivalent to SIT [29]. In Table 1 a survey on SRS, TPP and DTPP is shown together with some structural relationships to SIT (left) and SHG (right). In the graphs horizontal lines represent atomic levels while vertical lines symbolize field frequencies. In the lower part of the Table the equations of motion are written for two steps of reduction. *Real amplitudes* means exact resonance and pure amplitude modulation. χ and τ are space and retarded time variables respectively.

5. Three levels and two (or three) waves

For three levels in Λ-, V- or cascade configuration there is a rich field of phenomena [30-35]. S-integrability is established under some restriction for the coupling constants [36–38]. In particular this holds for some type of degenerate SIT where soliton solutions containing internal polarization waves were established [39].

6. Four waves

A particular type of 4-wave interaction is mathematically equivalent to SRS [40–42]. Finally we mention a few books covering solitons in nonlinear optics [42–44].



	SIT	SRS	TPP	DTPP	SHG
REAL AMPLITUDES	$\psi_{\chi\tau} = \sin\psi$	$\varphi_\chi = \sin\psi$ $\psi_\tau = \sin\varphi$	$\varphi_\chi = \sin\psi$ $\psi_\tau = \sinh\varphi$	$\varphi_\chi = \sin\psi$ $\psi_\tau = e^\varphi$	$\varphi_{\chi\tau} = e^\varphi$
WEAK EXCITATION	$\psi_{\chi\tau} = \psi$	$\varphi_{\chi\tau} = \sin\varphi$	$\varphi_{\chi\tau} = \sinh\varphi$	$\varphi_{\chi\tau} = e^\varphi$	$\varphi_{\chi\tau} = \varphi$

Table 1. Second–order resonance processes with relationships to SIT and SHG

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