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Notes From the SLAC Theory Workshop on the ¥

Concerning:
Interference Effects
Angular Distributions
How to Extract Widths
Radiative Corrections

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NOTE: The numbers quoted herein are NOT to be quoted in publication. They are meant to be illustrative, not definitive.

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This document is a working paper on the phenomenology of the $\phi$, which consists of two principal parts. It has evolved during a series of seminars on the $\phi$ with broad participation of the theoretical and experimental community at SLAC. The first part is a brief synopsis of the main tests and conclusions which can be extracted from the data, and is concerned with results. The second part is a collection of more detailed discussion of various points of the first section, and some independant discussions.

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B. Spin > 1 (G. Ringland, D. Wright)

If the \( \psi \) couplings to leptons preserves chirality then

\[
\frac{d\sigma}{d\Omega} = A(s) \left| d_{J/1}^J(\Theta) \right|^2 + B(s) \left| d_{J/1}^{J-1}(\Theta) \right|^2
\]

\( e^+e^-\rightarrow \mu^+\mu^- \)

If \( C \) and \( P \) are conserved, then \( A(s) = B(s) \). The distributions are plotted for \( J = 2, 3, 9, 10 \). They are self explanatory. (Figs. 2-3).

The most general analysis for spin \( J \) is too cumbersome to be informative.

C. Spin 1: (Budny, Cvitanovic, Giles, Pearson...)

If there exists no CP violation and no anomalous moment couplings, the \( \psi \) contribution to the \( e^+e^-\rightarrow \ell\ell \) amplitude can be written:

\[
\frac{\bar{\epsilon}_e(p') \left[ g_V \gamma^\mu + g_A \gamma^\mu \gamma^5 \right] \bar{\nu}_e(p)}{s-m_\ell^2+i\epsilon} \frac{1}{\epsilon} \frac{\bar{\nu}_e(k) \left[ g_V \gamma^\mu + g_A \gamma^\mu \gamma^5 \right] \nu_e(k')}{s-m_\ell^2+i\epsilon}
\]

where \( g_V \) and \( g_A \) are real.

This gives differential cross sections

\[
\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{s}{(s-m_\ell^2)^2 + m_\ell^2\pi^2} \left[ (g_V^2 + g_A^2)(1 + \cos^2\Theta) + 8g_V^2g_A^2 \cos\Theta \right]
\]

\[
+ \frac{\alpha}{8\pi} \frac{s-m_\ell^2}{(s-m_\ell^2)^2 + m_\ell^2\pi^2} (g_V^2 + g_A^2) \frac{(1 + \cos^2\Theta)^2}{1 - \cos^2\Theta}
\]
(Fig. 2)

(Fig. 3)
\[
\frac{d\sigma}{d\omega} = \frac{1}{64\pi^2} \frac{S}{(S-m^2)^2+m^2\pi^2} \left[ \left( q^2 + g_A^2 \right)^2 (1+\cos^2\Theta) + 8g^2 q_A^2 \cos\Theta \right] + \frac{\alpha}{8\pi} \frac{S-m^2}{(S-m^2)^2+m^2\pi^2} \left[ q^2 (1+\cos^2\Theta) + 2g^2 q_A^2 \cos\Theta \right] + \frac{\alpha^2}{45} (1+\cos^2\Theta)
\]

\[e^+e^- \rightarrow \mu^-\mu^+\]
where we have assumed universality of electron and muon couplings. (If not:
\[ g_V^2 = g_V(e)g_V(\mu) \text{ and } g_A^2 = g_A(e)g_A(\mu). \]

The interference terms between \( \psi \) and the photon are reflected in the behavior of the total cross sections and angular distributions:

**Total Cross Sections**

The interference terms cause dips in the total cross sections for both \( e^+e^- \) and \( \mu^+\mu^- \). For \( \mu^+\mu^- \) the dip is a near zero of the theoretical cross section on the low energy side of the peak.

\[
M - E_{\text{dip}} \approx \frac{3}{2\alpha} \left( \frac{g_V^2 + g_A^2}{q_V^2} \right) M \Rightarrow e^+e^- \approx 1 \text{ MeV for } \gamma(3105)
\]

There is a corresponding enhancement on the high energy side. After smearing by the beam resolution (fig. 4)

\[
M - E_{\text{dip}} \approx 3 \text{ MeV for } \gamma(3105), \quad \frac{\sigma_{\text{min}}}{\sigma_{\text{background}}} \approx 50\%-70\%
\]

The presence of the interference dip in \( e^+e^- \rightarrow \mu^+\mu^- \) implies that \( \psi \) cannot have pure axial couplings.

The sign of the effect in \( \sigma_{e^+e^- \rightarrow e^+e^-} \) is reversed and markedly less pronounced. \( E_{\text{dip}} \lesssim M \) because the predominant interference is with the t-channel photons cancelling the interference with the s-channel photons (Fig. 5). Any such dip will be buried under the radiative tail on the high energy side of the peak.

It is worth mentioning that experimentally one need not measure the cross section at the dip where event rates are low in order to see the interference terms. Any measure of the skewness of the total cross section relative to a pure Breit-Wigner near the peak will suffice.
$\sigma_{e^+ e^- \rightarrow \mu^+ \mu^-}$

$\Gamma_T = 92$ KeV

$\Gamma_1 = 5.5$ KeV

(Fig. 4)
\( \sigma_{e^+e^-\to e^+e^-} \)

\( \Gamma_T = 92 \text{ KeV} \)

\( \Gamma_L = 5.5 \text{ KeV} \)

(Fig. 5)
changes by a factor $\sim 2$ over the region $m_\psi - 1, \text{MeV} \rightarrow m_\psi + 1, \text{MeV}$.

**Angular Distributions**

$e^+ e^- \rightarrow \mu^+ \mu^-$

Any large parity violation in the $\psi$ itself ($g_V \sim g_A$) is observable as a front-back asymmetry at the peak in $e^+ e^- \rightarrow \mu^+ \mu^-$

$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta)^2$$

(Fig. 6)

For nearly pure $V$ or $A$ the distribution at the peak is $(1 + \cos^2 \theta)$, so one must look in the interference region to distinguish the two cases.

For pure vector the angular distribution is $1 + \cos^2 \theta$ at all energies — there is no front-back asymmetry.

For purely axial vector $\psi$, there is a front-back asymmetry

$$A \equiv \frac{\sigma_{50^\circ < \theta < 90^\circ} - \sigma_{90^\circ < \theta < 135^\circ}}{\sigma_{50^\circ < \theta < 135^\circ}}$$

that is negative in the interference region and positive in the radiative tail of magnitude $\sim 35\%$. (Fig. 6)
Front-Back Asymmetry $e^+e^- \rightarrow \mu^+\mu^-$

Asymmetry = \frac{\sigma_{\phi > \phi_0} - \sigma_{\phi < \phi_0}}{\sigma_{\phi > \phi_0}}$

(Fig. 6)
For the $e^+e^-$ reaction, the information in the angular distributions is more difficult to extract. The background Bhabha process has a front-back asymmetry $A=0.66$. For $V=A$, this asymmetry is changed only slightly near the peak (Fig. 7). For either pure $V$ or pure $A$, $A$ is decreased at the peak by the addition of a large $(1+\cos^2\theta)$ term. The effects in the interference region are relatively small.

Is there more than one $\psi$ (or $\psi'$)?

Various theories could have more than one neutral $\psi$, conceivably degenerate in mass to an MeV. (See Barshays preprint; also colored $\rho^0$ degenerate with colored $\omega^0$ is another such option.) A variety of interference effects are possible, depending on whether the two lines overlap, one is broad, one narrow, etc. The consequences for experiment are

(i) Branching ratios, distributions, etc. on the high side of the resonance may differ from those on the low side, and

(ii) The line shapes may be peculiar.

A general study for the lepton channels, assuming two spin one $\psi$'s, is given by B. Ward.

References in the second part for this section are:

Effects of P and C symmetry on angular distributions ...
A. Weldon, S. Brodsky, J. D. Jackson, and J. Kuhn

Two $\psi$'s .... B. Ward

Calculations of crosssections .... R. Budny, and R. Giles
(Fig. 7)