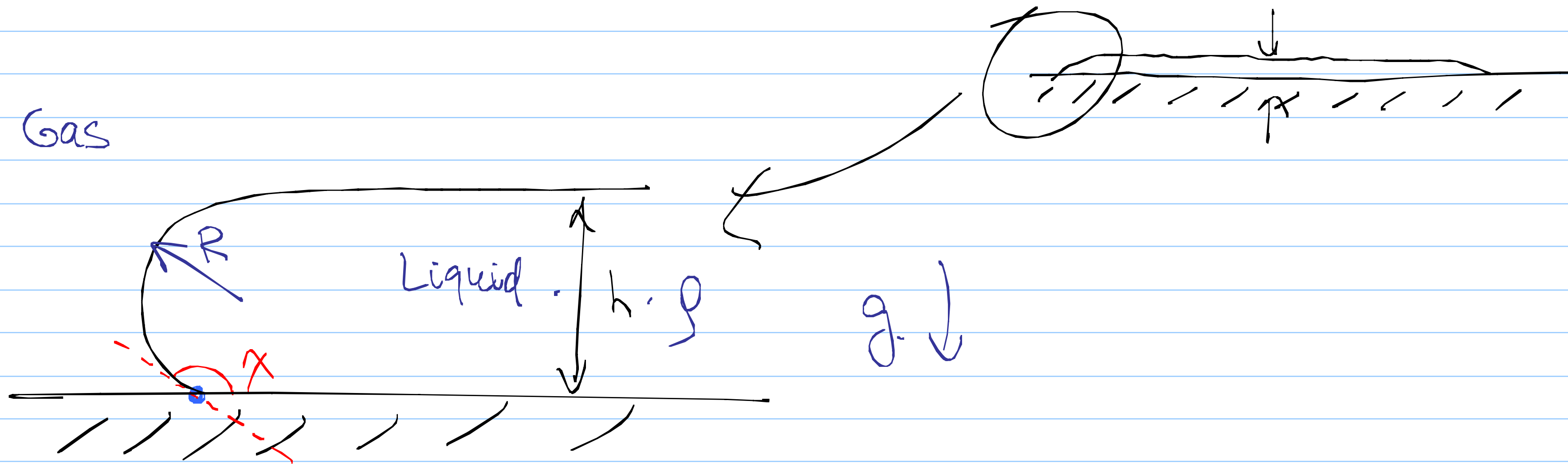
Capillarity & Contact angles

Note Title 10/31/2008

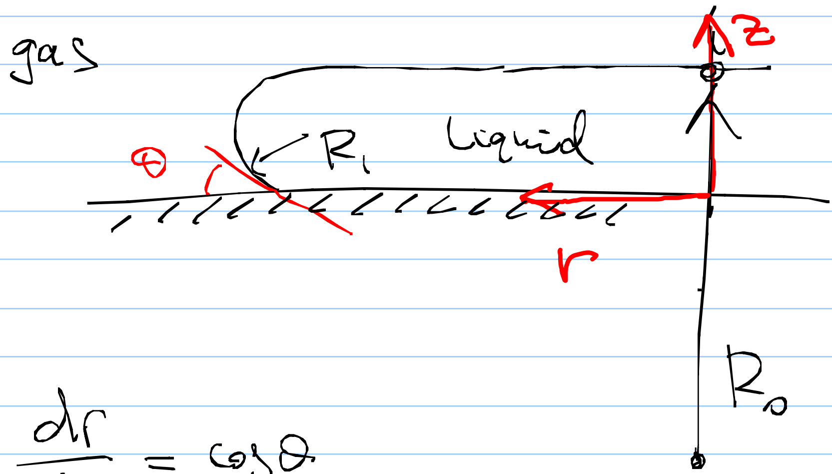


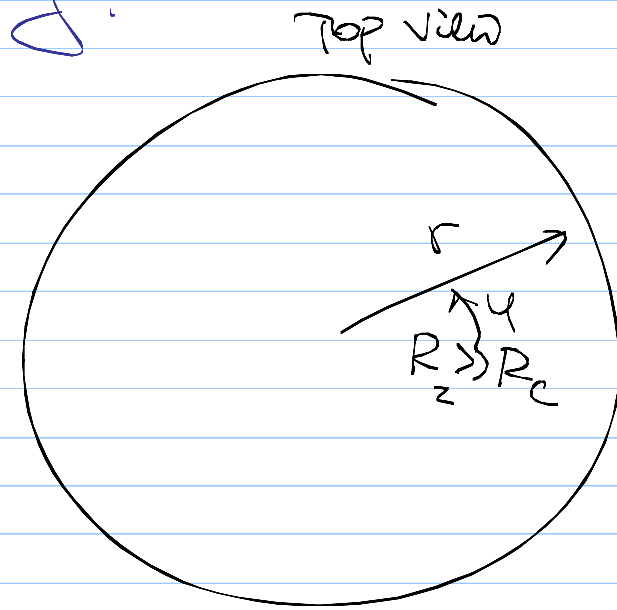
$$\langle p \rangle = \frac{\rho g h}{2}, \quad P_{cap} = \frac{\alpha}{R}, \quad h = R(1-\cos\chi)$$

$$P_{cap} = \langle p \rangle \;\rightarrow\; \frac{\rho g h}{2} = \frac{\alpha}{h}(1-\cos\chi)$$

$$\Rightarrow h^2 = \frac{\alpha}{\rho g}\, 2(1-\cos\chi) \;\Rightarrow\; h = R_c\sqrt{2(1-\cos\chi)}$$

Now do the calculation properly:





$$\frac{dr}{ds} = \cos\theta$$

$$\frac{dz}{ds} = \sin\theta$$

$$\frac{d\theta}{ds} = \frac{2}{R_0} - \frac{\sin\theta}{r} + \frac{z}{R_c^2}$$

← hydrostatic pressure

curvature in (r,φ) plane, $\frac{1}{R_2}$

"free" pressure normalization

curvature in (r,z) plane, $\frac{1}{R_1}$

$$\frac{dr}{d\theta} = \frac{dr}{ds}\cdot\frac{ds}{d\theta} = \frac{\cos\theta}{\frac{z}{R_c^2} - \frac{\sin\theta}{r} + \cancel{\frac{2}{R_0}}}$$

$$\frac{dz}{d\theta} = \frac{dz}{ds}\cdot\frac{ds}{d\theta} = \frac{\sin\theta}{\underbrace{\frac{z}{R_c^2}}_{O\left(\frac{1}{R_c}\right)} - \underbrace{\cancel{\frac{\sin\theta}{r}}}_{O\left(\frac{1}{R_c}\right)} + \underbrace{\cancel{\frac{2}{R_0}}}_{O\left(\frac{1}{R_0}\right)}} \simeq R_c^2\cdot\frac{\sin\theta}{z}$$

$$O\left(\frac{1}{R_c}\right) \ll O\left(\frac{1}{R_c}\right) \ll O\left(\frac{1}{R_0}\right)$$

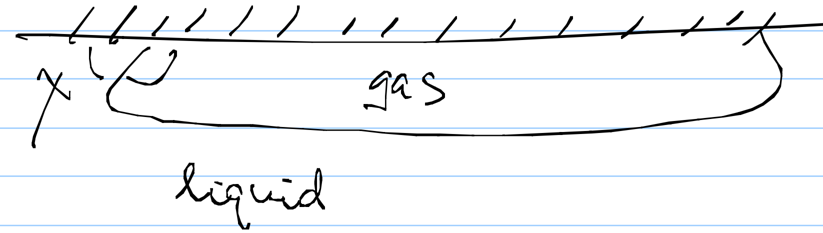
$$\rightarrow \int_0^h z\cdot dz = \int_{\pi-\chi}^{\pi} R_c^2 \sin\theta\, d\theta \rightarrow \frac{h^2}{2} = R_c^2(-\cos\theta)\Big|_{\pi-\chi}^{\pi} = R_c^2(1-\cos\chi)$$

$$\Rightarrow h = R_c\sqrt{2(1-\cos\chi)}$$

Accuracy: $\frac{\delta h}{h} \sim \frac{\frac{1}{r}}{\frac{z}{R_c^2}} \sim \frac{\frac{1}{R_2}}{\frac{1}{R_c}} \sim \frac{R_c^{1/2}}{V^{1/2}}\cdot R_c = \frac{R_c^{3/2}}{V^{1/2}} = \left(\frac{R_c^3}{V}\right)^{1/2}$

$$V \sim \pi R_2^2 \cdot R_c \Rightarrow R_2 \sim \left(\frac{V}{R_c}\right)^{1/2}$$

<u>Note</u>: $h = R_c\sqrt{2(1\pm\cos\chi)}$



<u>Example</u>:

$$2\cdot \cancel{L}\alpha\cdot\sin\theta = Mg = \rho\pi a^2 \cancel{L}\cdot g$$

$$a \le \frac{2}{\pi}\frac{\rho g}{\alpha} = \frac{2}{\pi}R_c^2 \Rightarrow a \le \sqrt{\frac{2}{\pi}}R_c$$

$$\sin\theta = \frac{\pi a^2 \rho g}{2\alpha} = \frac{\pi}{2}\left(\frac{a}{R_c}\right)^2$$

$$h = R_c\sqrt{2(1-\cos\theta)} \sim h(a, R_c)$$