Problem 1

Use the Cauchy-Reimann condition to determine whether the following function is analytic:

(a) $w = |z|^2$
(b) $w = e^z$
(c) $w = \sin z$

Problem 2

Determine how the following functions $w = f(z)$ map regions in the complex $z$-plane to regions in the complex $w$-plane:

(a) Circle $|z| < a$, half-plane $\text{Re}(z) > 0$, half-plane $\text{Im}(z) > 0$ under $w = \frac{z-a}{z+a}$, where $a$ is real and positive.
(b) Lines $\text{Re}(z) = \text{const}$, the strip $-\frac{\pi}{2} < \text{Re}(z) < \frac{\pi}{2}$ under $w = \sin z$. 