Problem 1

Bugatti Veyron needs a bigger engine to reach the claimed 255 mph top speed than, say, a Toyota Corolla, which has a 132 hp engine and a 115 mph top speed. The top speed is limited by air resistance, which exerts a force

\[ F = \frac{1}{2} C_d \rho v^2 A \]  

on the car, where \( \rho \) is the density of air, \( A \) is the cross-section of the car, and \( C_d \) is the drag coefficient. \( C_d \) is about the same for both cars (about 0.3) and \( A \) is also comparable. Use scaling analysis to determine the minimal power a Bugatti Veyron’s engine needs to develop to reach its top speed.

Problem 2

Determine how a rowboat’s speed \( v \) scales with the number \( N \) of rowers. The speed of the boat is also limited by (1), where \( \rho \) is now the density of water and \( A \) is the cross-sectional area of the submerged part of the boat.

1. Assuming that the boat is light compared to the rowers, compute how \( A \) scales with \( N \).
2. Using this scaling, compute how \( v \) scales with \( N \).

Problem 3

Determine how much power Yoda can produce. Yoda’s greatest display of raw power in the original Star Wars trilogy came when he lifted Luke Skywalker’s X-Wing from a swamp on the planet Dagobah.

1. First we need to estimate the X-Wing’s mass. It has never been canonically established, but the X-Wing’s length has been: 12.5 meters. Use scaling to estimate the mass \( m \) of the X-Wing by comparing it to something similar – an F-22 fighter plane.

2. Next we need to estimate the speed \( v \) with which the X-wing is lifted. The front landing strut rises out of the water in about three and a half seconds, and its length can be estimated to be about 1.4 meters long (based on a scene in A New Hope where a crew member squeezes past it).
3. Finally, according to Wookiepedia, Dagobah has a diameter of 8900 km and we can reasonably estimate that its density is comparable to that of the Earth. Estimate the gravitational acceleration $g$ on Dagobah and use this value, together with the X-Wing’s mass $m$ and speed $v$, to compute the power of Yoda in units of horsepower.

**Problem 4**

The “radiative power”, or the energy radiated away by a “black body” per unit surface area per unit time, is given by the Stefan-Boltzmann law

$$ j = \sigma T^4, \quad (2) $$

where $\sigma = 5.67 \times 10^{-8}$ Js$^{-1}$m$^{-2}$K$^{-4}$ is the Stefan-Boltzmann constant (you don’t need the numerical value for this problem) and $T$ is the surface temperature in Kelvin.

1. Let us ignore the absorption/reflection of radiation in the atmosphere (aka the green-house effect) and the heat generated by radioactivity in the core and assume that the temperature of the Earth (at the surface) is solely due to the balance between the radiative energy received from the Sun and that lost to space. Based on this information, use scaling arguments to compute the temperature of the surface of the Sun, which has a radius of $R_s = 7 \times 10^8$ m and is one astronomic unit ($L_{\text{earth}} = 1.5 \times 10^{11}$ m) away from the Earth.

2. Using scaling, compute the average temperature on the surface of Mercury ($L_{\text{mercury}} = 0.4$ AU), Venus ($L_{\text{venus}} = 0.7$ AU), and Mars ($L_{\text{mars}} = 1.52$ AU).

**Problem 5**

Let’s compute the radii of Bohr’s orbits using the semiclassical quantization condition $S = nh$, where $n$ is a positive integer, $h$ is the Plank constant,

$$ S = \oint p \, ds \quad (3) $$

is the classical action, $p = mv$ is the momentum of the electron, and $s$ is the arclength of its trajectory around the nucleus (which we will assume is stationary). For a circular orbit $ds = r \, d\theta$, so $S = mv \times 2\pi r$.

1. Derive “Kepler’s 3rd law” for the motion of electron(s) around the nucleus of charge $Ze$, including the expression for the proportionality constant.

2. Using this law and the classical quantization condition, derive the expression for the radii $r_n$ of Bohr’s orbits.

3. By what factor would $r_1$ change if we were to replace the electron in the hydrogen atom with a muon?

4. By what factor would $r_1$ change, if we replaced the electron in the Hydrogen atom with a $\bar{u}$ quark? Be careful here, as quarks carry a different charge. Feel free to use Wikipedia to look up the properties you don’t know off the top of your head.