Phys4803/8803: Homework set #3

Problem 1 (Kepler’s 3rd law)
Derive Kepler’s 3rd law of planetary motion using dimensional analysis.

Problem 2 (Wind turbine)

1. Use dimensional analysis to find the most general expression for the power $P$ produced by a wind turbine with rotor blades of length $r$, if the wind speed is $v$.

2. Suppose $r = 40$ m and the wind speed is $v = 30$ m/s. Estimate the Reynolds number and decide whether $P$ can depend on the viscosity and density of air. Using the appropriate limit of the expression from part 1, estimate the power generated by the turbine.

Problem 3 (Light bending)

1. Use dimensional analysis to estimate the angle $\theta_p$ by which light is deflected when passing at a distance $r$ from a star of mass $M$, yielding a functional relation $\theta_p = f(\Phi)$, where $\Phi$ is independent of $\theta_p$.

2. For light that we can observe here on earth, $\theta_p$ has to be very small. Since gravitational field produced by mass $M$ is linear in $M$, $\theta_p$ should be a linear function of $M$. Using this fact, simplify your result from part 1.

3. Compare your result with the deflection angle $\theta_n$ for an ultra-relativistic neutron (which, unlike photons, has mass). Compute $\theta_n$ explicitly, assuming the angle of deflection is small and the speed of neutron is close to the speed of light (but not using relativistic formulas). The answer coincides with the deflection angle $\theta_p$ for a photon, and also gives a numerical prefactor.

   Hint: To compute $\theta_n$, integrate the equations of motion in a plane passing through the star and containing the trajectory of the neutron. Use Cartesian coordinates with axis $x$ oriented along the initial direction of motion.

Problem 4 (Bohr’s model of Hydrogen)

1. Let’s determine Bohr’s radius $a_0$ as a function of the electron’s charge $e$ and mass $m_e$ and various fundamental constants. Since we are dealing with quantum mechanics, we clearly need to include the Planck constant $\hbar$ and, since the charges are bound electrostatically, we need $\epsilon_0$ (which is a conversion factor between electrical and mechanical units). Write down the most general expression for $a_0$ as a function of $e$, $m_e$, $\hbar$, and $\epsilon_0$.

2. Repeat this exercise to find the expression for the Rydberg energy (the ground state energy of the Hydrogen atom).
Problem 5 (Black body spectrum)

Black body radiation spectrum is determined by the Planck’s law

\[ u = \frac{2h\nu^3}{c^2} \left[ \exp\left( \frac{h\nu}{k_B T} \right) - 1 \right]^{-1}, \]  

where \( u \) is the power radiated per unit area per unit frequency.

1. Determine the nondimensional combinations of \( u, h, c, \nu, k_B \), and \( T \) and use them to rewrite Planck’s law in nondimensional form \( \Pi_1 = f(\Pi_2) \). What is the explicit form of \( f(\Pi_2) \)?

2. The Excel file [http://www.cns.gatech.edu/~roman/phys8803/planck.xls](http://www.cns.gatech.edu/~roman/phys8803/planck.xls) contains the normalized radiation spectra from several different objects: a supernova, our sun, and a planet (Pluto). Scale the intensity and the frequency such that the points for all three data sets collapse onto the Planck’s curve. Using these scalings determine the surface temperature of each object.

*Hint: You may want to use Excel to reduce the manual labor involved in rescaling of the data sets.*