Phys4803/8803: Homework set #5

Problem 1 (Higher orders of perturbation theory)
Compute the perturbative solutions for both roots of the quadratic equation
\[ x^2 + \epsilon x - 1 = 0 \]  
with \( \epsilon \) small. Compute correction up to terms of order \( \epsilon^5 \).

*Hint: You are encouraged to use Maple or Mathematica to do the algebra in this assignment. As you will find, the algebra can get out of hand rather quickly, if you do it manually.*

Problem 2 (Singular perturbation expansion)
Compute the perturbative solutions for all three roots of the cubic equation
\[ \epsilon x^3 - x^2 + 1 = 0 \]
with \( \epsilon \) small.

1. Find the regular root(s) using regular perturbation expansion up to terms of order \( \epsilon^2 \).
2. Use scaling and dominant balance to compute the singular root(s) up to terms of order \( \epsilon \).

Problem 3 (Nonintegral powers)
Compute the perturbative solution to the cubic equation
\[ x^3 - 3x^2 + 3x - 1 = \epsilon \]
with \( \epsilon \) small.

1. Using the regular perturbation expansion
\[ x = x_0 + \delta x_1 + \delta^2 x_2 + \cdots \]
with \( \delta = \epsilon \), show that it is impossible to find an expansion in integral powers of \( \epsilon \).
2. Use dominant balance to compute \( \delta(\epsilon) \). How many different possibilities do you get?
3. Express \( \epsilon \) in (3) in terms of \( \delta \) and compute the solution perturbatively up to terms of \( O(\delta^2) \).

Problem 4 (Electrical circuit, again)
Recall the equation for an electrical circuit from Problem 4 in the previous assignment
\[ L\ddot{Q} + R\dot{Q} + C^{-1}Q = 0. \]
1. Use the method of dominant balance to obtain all approximate solutions of this equation in the various limits in dimensional form. (In nondimensional form those correspond to equations (7)-(9) from the previous assignment with $\epsilon \to 0$).

2. Extract the relevant time scale $s_T$ from each of the dimensional solutions.

3. Explain what physical problem each of these limits corresponds to and compare the time scale $s_T$ you extracted from the dimensional solution in part 2 with the corresponding time scale obtained using dimensional analysis in the previous assignment.