Homework 1

1. For total length, we will need

   \[ \text{Length of an avg hair} = \frac{\text{Number of hairs}}{\text{Unit area of head}} \cdot \text{area covered by hair} \]

   The average woman is about 1.5 m tall and the average hairstyle
   reaches her shoulders about \( \frac{2}{3} \) of her height, so
   \[ \text{Average hair length} = \frac{3}{2} \times 1.5 = \frac{9}{2} \text{ m} \]

   Looking at a friend's scalp, it looks like one cm of length gets to about
   10 hours, so 1 m² unit area will have \( 10^6 \) hours

   We can approximate an average woman's head as a sphere with a radius
   of about 10 cm, if half of it is covered in hair,
   \[ A = \frac{1}{2} \cdot 4 \pi (0.1 \text{ m})^2 = 6 \times 10^{-3} \text{ m}^2 \]

   So total length: \( L = \frac{9}{2} \text{ m} \cdot 10^6 \text{ hours/m}^2 \cdot 6 \times 10^{-3} \text{ m}^2 = 1.5 \times 10^4 \text{ m} \]

   The surface area of all this hair would be the length times the cross-sectional
   area, if an average hair has a radius of \( \approx 0.1 \text{ mm} = 10^{-4} \text{ m} \)
   \[ A = \pi (10^{-4} \text{ m})^2 \cdot 2 \pi (10^{-4} \text{ m})(1.5 \times 10^4 \text{ m}) \approx 9 \text{ m}^2 \]

2. For this problem we need the quantities

   Research physicists in the world: Annual productivity of one physicist
   None of which I know to any degree of accuracy.

   To start with the total physicists, I heard one that 200 physicists enter
   academic in the US each year. If they remain for their whole
   careers, say 40 years, that's about 8000 in the US.

   The portion of physics happening in the US can't be strictly proportional to
   population. GDP would probably be a better indicator, and if the
   US has \( \frac{1}{4} \) the world's total GDP, that leaves 4 \( \times \) 2000 = 3.2 \( \times \) 10^4 physicists.

   Productivity probably varies a lot, but it's reasonable to say it's typically between
   1 and 10 papers per year for a geometric mean of \( \sqrt{10} \approx 3 \text{ papers/yr} \)
   \[ \text{Papers/yr} = 3 \times 10^4 \text{ physicists} \cdot 3 \text{ papers/yr/physicist} = 10^5 \text{ papers/yr} \]
2) How far does it take to strip of 1 molecule of rubber?
   1) How much is thickness difference between new and old tires?
   2) How long does it take (in km) to wear down tires
   1) New tire tread depth is about 1 cm
   2) Old tire tread depth is about .2 cm
      \[ \Delta D = 1 - .2 = .8 \text{ cm/cycle} \]
   2) Tires last for about 50,000 miles = \( 80,000 \text{ km} \)
      
      1 atom thick is about 1 cm thick layer.
      
      Rate of stripping: \( \frac{80,000}{.8} = 100,000 \text{ km/cm} = 100,000,000 \text{ km/m} \)
      \[ D = 10,000,000 \times 10^{-3} = .01 \text{ km} \times 10 \text{ m} \]
Problem 4. What is (a) the kinetic energy and (b) the momentum of a drifting continent? Is this large or small?

Continental drift is occurring at a rate of 2 cm/yr in m/s, this is:

\[
\text{rate} = \frac{0.02 \text{ cm}}{\text{yr}} \times \frac{1 \text{ yr}}{365 \text{ days}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{1 \text{ min}}{60 \text{ min}} \times \frac{60 \text{ s}}{1 \text{ min}} = 0.6 \times 10^{-9} \text{ m/s}
\]

Then to get the mass of the continent: I'm assuming that this really means the tectonic plates, not so much the continent (where does a continent end? How deep is a continent?... tectonic plates are clearly defined...). Then, the Earth is completely covered by 9 plates (7 continental plates + Pacific plate; there are also minor plates but we will ignore them). The surface area of the Earth is:

\[
A = \pi (R^2) \approx 3 \times (6400 \times 10^3 \text{ m})^2 = 1 \times 10^{14} \text{ m}^2
\]

Then the surface area of 1 plate is:

\[
\frac{10^{14} \text{ m}^2}{9 \text{ plates}} \approx 10^{13} \text{ m}^2/\text{plate}
\]

Then a plate is as thick as the Earth's crust, which we'll estimate to 30 km (bracketing with 10 km and 100 km). Then the volume of a plate is:

\[
\frac{10^{13} \text{ m}^2}{\text{plate}} \times 30 \times 10^3 \text{ m} = 3 \times 10^{17} \text{ m}^3/\text{plate}
\]

Then estimate the density of rock to be roughly the density of water (which is fine to order of magnitude), or 1.9/cm^3 = 1.9 \times 10^3 \text{ kg/m}^3. So the mass of a continent is:

\[
3 \times 10^{17} \text{ m}^3/\text{plate} \times 10^3 \text{ kg/m}^3 = 3 \times 10^{20} \text{ kg/continent}
\]

Then the kinetic energy of a plate/continent is:

\[
K = \frac{1}{2} mv^2 = \frac{1}{2} (3 \times 10^{20} \text{ kg})(6 \times 10^{-10} \text{ m/s})^2 = \frac{1}{2} (3 \times 10^{20})(3 \times 10^{-20}) \text{ J}
\]

And the momentum is:

\[
p = mv = (3 \times 10^{20} \text{ kg})(6 \times 10^{-10} \text{ m/s}) = 18 \times 10^{10} \text{ kg m/s}
\]

The energy is not THAT high, but the momentum is huge (which makes sense, because it should be very hard to stop a continent)
5) 1) What is the momentum of a 6 car train?

A subway car weighs more than 10,000 kg and less than 100,000 kg.

\[ \sqrt{1000 \cdot 10000} \approx 30,000 \text{ kg} \]

6 cars \( \approx 180000 \text{ kg} \)

It moves between 30 mph and 100 mph.

\[ \sqrt{30 \cdot 100} = 55 \text{ mph} = 80.4 \text{ km/h} \approx 22.9 \text{ m/s} \]

\[ \Delta P = \frac{mv}{2} = 180000 \cdot 22.9 = 4000000 \text{ km} \]

2) How long does it take the train to stop?

1 block \( \approx 100 \text{ m} \)

15 blocks \( \cdot 100 \text{ m} = D = 1500 \text{ m} \)

Assuming a constant acceleration:

\[ \Delta t = \frac{D}{(V_{max}/2)} = \frac{1500}{62.5} = 120 \text{ seconds} \]

\[ \text{Force} = \frac{\Delta P}{\Delta t} = \frac{4,500,000}{120} = 37500 \text{ N} = 3.75 \text{ Tons} \]

I can lift about 0.75 tons. Spiderman is so strong.

Stronger than me.