# Dynamical systems approach to 1-$d$ spatiotemporal chaos – A cyclist's view

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# Abbreviations and Synonyms

SIS: Strange Invariant Set, an invariant set that has a fractal dimension.
UPO: Unstable Periodic Orbit.
cycle: a periodic orbit.
PDE: Partial Differential Equation.
ODE: Ordinary Differential Equation.
KSe: Kuramoto-Sivashinsky equation.
CGLe: Complex Ginzburg-Landau equation.
NSe: Navier-Stokes equation

# SUMMARY

We propose a dynamical systems approach to study of turbulence (spatiotemporal chaos) based on the periodic orbit theory. The role of recurrent patterns and coherent structures is emphasized in the description. After a brief review of the periodic orbit theory in low dimensions, we discuss its possible application to the dynamics of spatially extended systems. The discussion is three-fold. First, we introduce a novel variational scheme for finding periodic orbits in general flows. Second, we check the periodic structures near the first instability of the complex Ginzburg-Landau equation and talk about its role in pattern formation. Third, we discuss the extensive numerical explorations that we have done to the Kuramoto-Sivashinsky system in the chaotic regime, including searching for periodic orbits, constructing symbolic dynamics and describing the chaotic invariant set in terms of intrinsic coordinates. In these investigations, we hope to identify and rank the nonlinear instabilities according to their relative importance.