CHAPTER VII

SUMMARY

Dynamical systems and turbulence have been a focus of research for centuries. Various approaches: numerical, analytical, algebraic, geometrical, statistical, topological, scaling, field theoretic, *etc.*, have been developed, and the advantagous stategy should combine all these methods. In a sense, periodic orbit theory is a strong candidate. It has been applied to many low-dimensional chaotic systems with a remarkable success. We are committed to generalizing this theory to study of spatiotemporal chaos (turbulence) in spatially extended systems. As a modest first step, here we have studied two $1-d$ PDEs on a compact domain with the periodic condition: the CGLe and the KSe.

The CGLe differs from the KSe in that it describes two interacting scalar fields while the KSe only depicts one. In the chaotic regime of the CGLe, topological defects appear and vanish at discrete space-time points, but the field in the KSe is always smooth. As the KSe can be derived as a phase equation from the CGLe [22], we may check the relationship of their dynamics and get a better understanding of both.

In the CGLe, we have investigated the onset of spatiotemporal chaos from a bifurcation point of view. Near the bifurcation point, the dynamics is regular or weakly chaotic, so analytical treatment is possible. We have also checked the role played by the invariant structures and tried to find their dynamical explanations. This system is an illumination of the idea that finitely many recurrent patterns may make up the key building blocks of the spatiotemporal chaos.

In the KSe, the structure of the SIS and the chaotic dynamics on it are explored using the periodic orbit approach. After checking the qualitative aspects of the dynamics, a complete set of UPOs up to some length is found and approximate symbolic dynamics established at the same time. Here, a variational method devised for high-dimensional unstable flows is applied for finding UPOs. Stationary points are found and their relation to the set of UPOs is studied, from which the qualitative structure of the phase space is dicussed. Finally, dynamical averages are calculated and typical patterns are picked up, which may be compared with the experimental or numerical observations.

The validity and applicability of the periodic orbit theory presupposes that the system is hyperbolic, so our calculatons depend much on the hyperbolicity assumption. Most systems in the interesting parameter regimes are not hyperbolic, and frequently intermittency plays an essential role. In some systems, the SIS and intermittency coexist. How to treat all these different cases and how to weigh different invariant sets in one dynamical system are open problems, important, and demanding real effort. The periodic orbit theory is making progress in this direction on 'simple' systems.

In long run, the hope is that we can apply the periodic orbit theory to real-world problems, such as the Navier-Stokes equation under given boundary conditions, and use calculated results to match or predict experimental data, or to check and modify the assumptions underlying specific turbulence models. More analytical tools for analyzing the invariant set and UPOs and more powerful numerical methods for detecting UPOs are waiting to be created, a task requiring a collaborative effort between people from many disciplines.